UNIVERSITY OF AMSTERDAM

Rick Quax, Andrea Apolloni, Drona Kandhai, Emiliano Mancini,
Peter M.A. Sloot

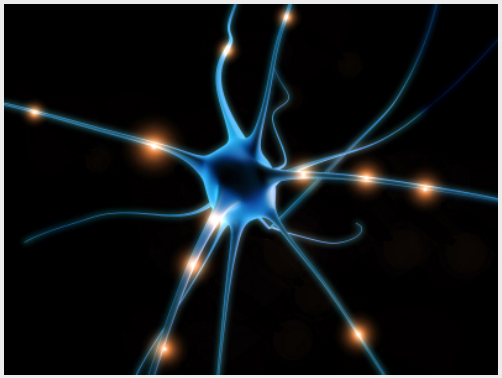
Information dissipation in networks

Information is stored in system state, transferred through interactions, and lost due to noise

Our view of a complex system

node dynamics + complex network = complex system

problem



+

=

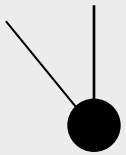
Each node has a state
which it changes over time

Nodes interact with each other
i.e., their states influence each other

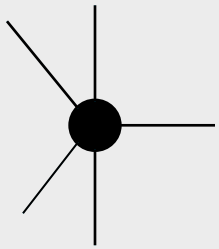
The system behavior is complex
compared to an individual node

Research question

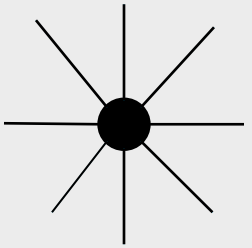
"Which units drive the behavior of the network?"



Peripheral units



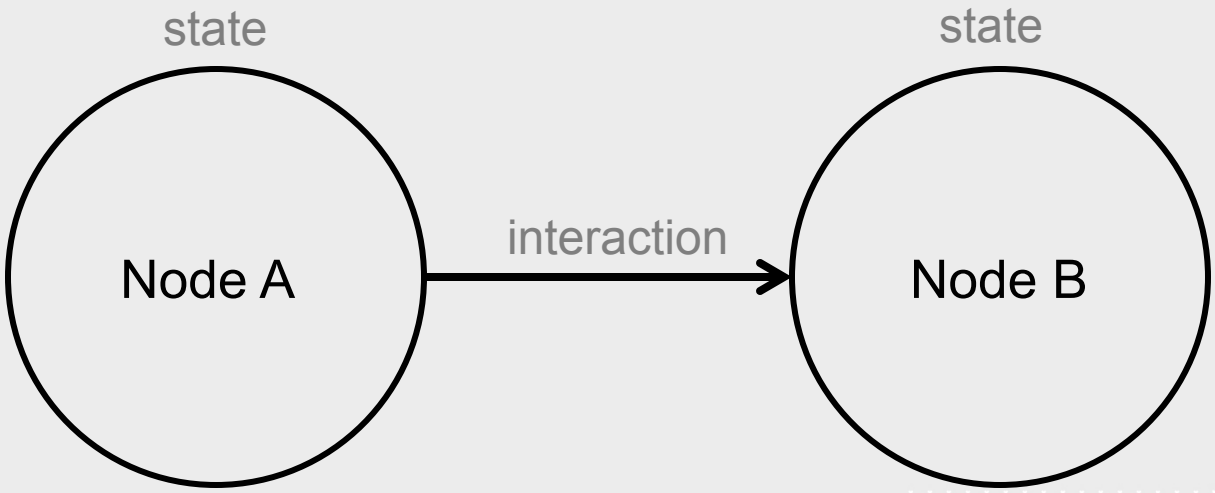
Intermediately connected units



Highly connected units

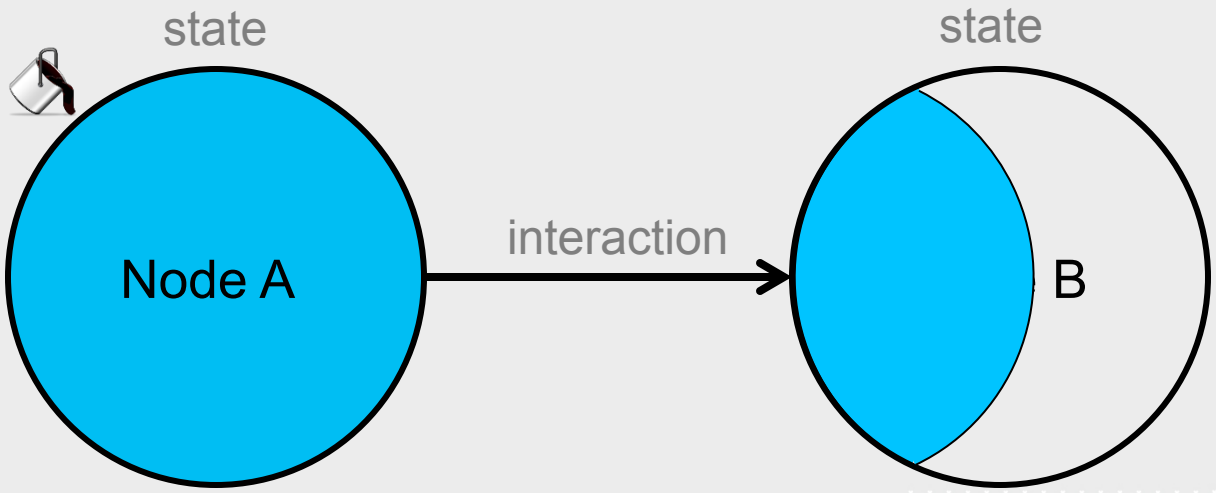
Information processing in complex systems

- Let's say the state of A influences the state of B…



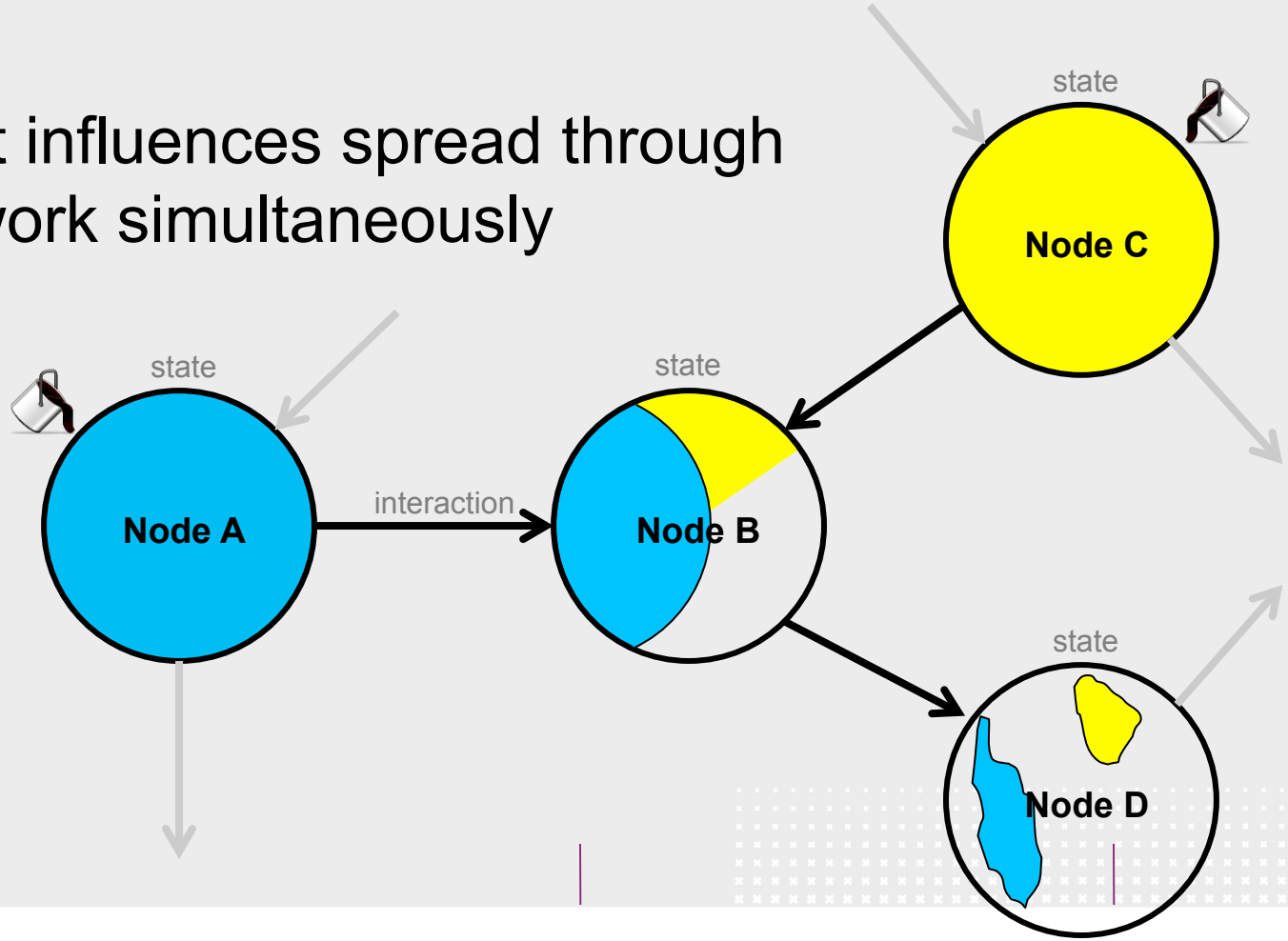
Information processing in complex systems

- We would like to ‘see’ influence spreading



Information processing in complex systems

- Different influences spread through the network simultaneously

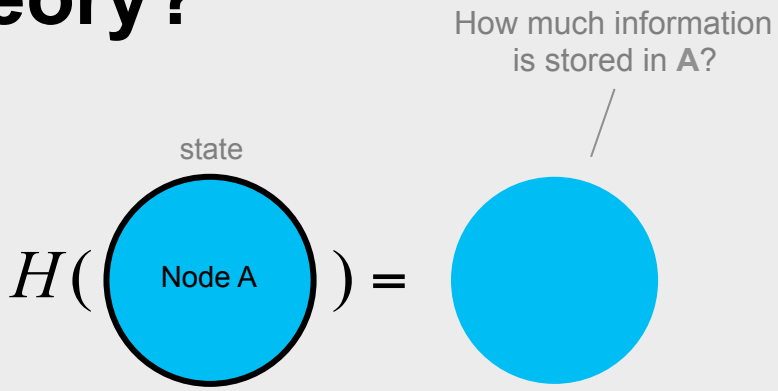


How to make make this quantitative?

Solution: information theory?

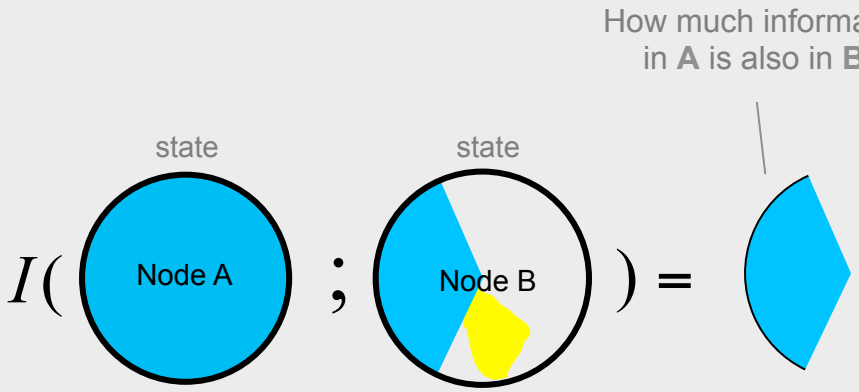
Entropy:

$$H(A) = -\sum_i p_{A=i} \log p_{A=i}$$



Mutual information

$$I(A;B) = H(B) - H(B \mid A)$$



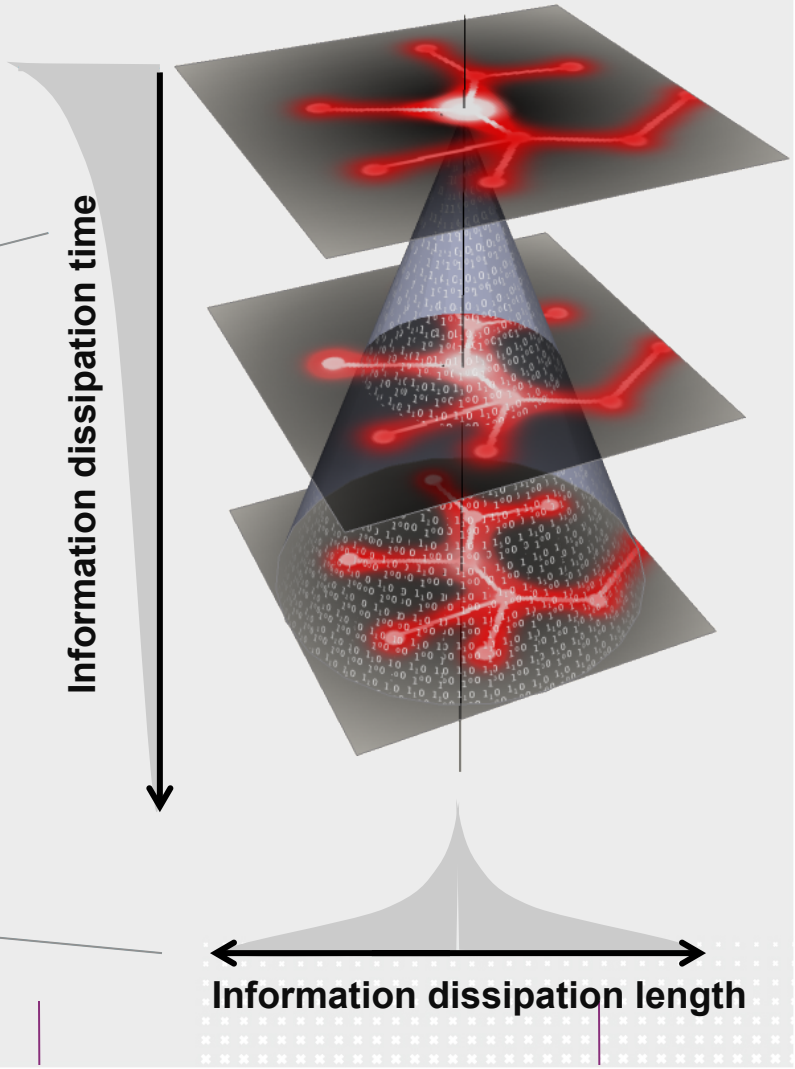
(pitfall: MI = causality + correlation)

Information dissipation

How long is the information about a node's state retained in the network?

measures of influence of a single node on the behavior of the entire network!

How far can the information about a node's state reach before it is lost?

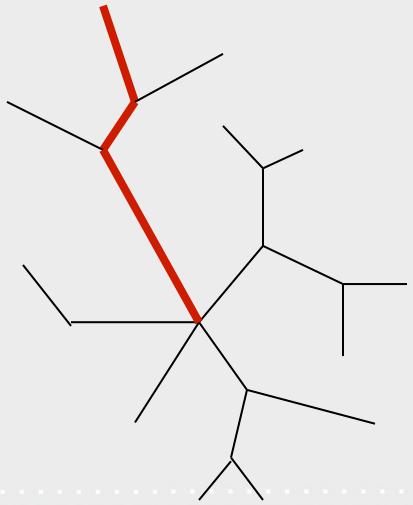


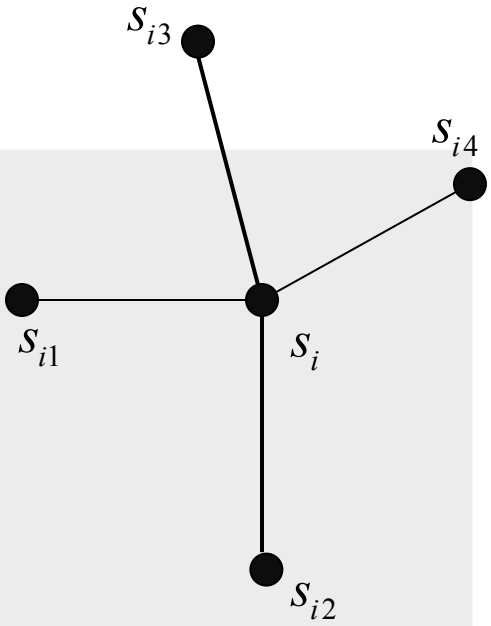
Interface
The diminishing role of hubs in dynamical processes on complex networks
Rick Quax[1↑], Andrea Apolloni[2,↑] and Peter M. A. Sloot[1,3,4]

Information dissipation time

$$p(s_i^{t+1} = x \mid s_j^t, \ldots) \propto \exp \sum_j -E(x, s_j^t)$$

- Edges represent an interaction potential to which a node can quasi-equilibrate
  - → Node dynamics: (local) Gibbs measure
- Network structure
  - Large
  - Randomized beyond degree distribution
  - $k_{\max}$ grows less than linear in N
    - And thus locally tree-like





Information dissipation time

$$I_0^k \equiv I(S^t; s_i^t) = I(s_i^t; s_i^t) = H(s_i^t)$$

$$I_1^k \approx I(\left[s_{j1}^t, \ldots, s_{jk}^t\right]; s_i^t)$$

$$\$ = \sum_m q(m) \cdot I_1^{m+1} / I_0^{m+1}.$$

$$D(s) = \log_{c_{\text{eff}} \cdot \$}\left[\frac{\varepsilon}{I_1^k}\right] = \frac{\log \varepsilon - \log I_1^k}{\log c_{\text{eff}} + \log \$}.$$

$$D(s) \propto \text{const} + \log I_1^k,$$

$$I_1^k = U(k) \cdot k \cdot T(k), \text{ where}$$

$$T(k) = \left\langle I(s_j^{t+1}; s_i^t) \right\rangle_{k_j},$$

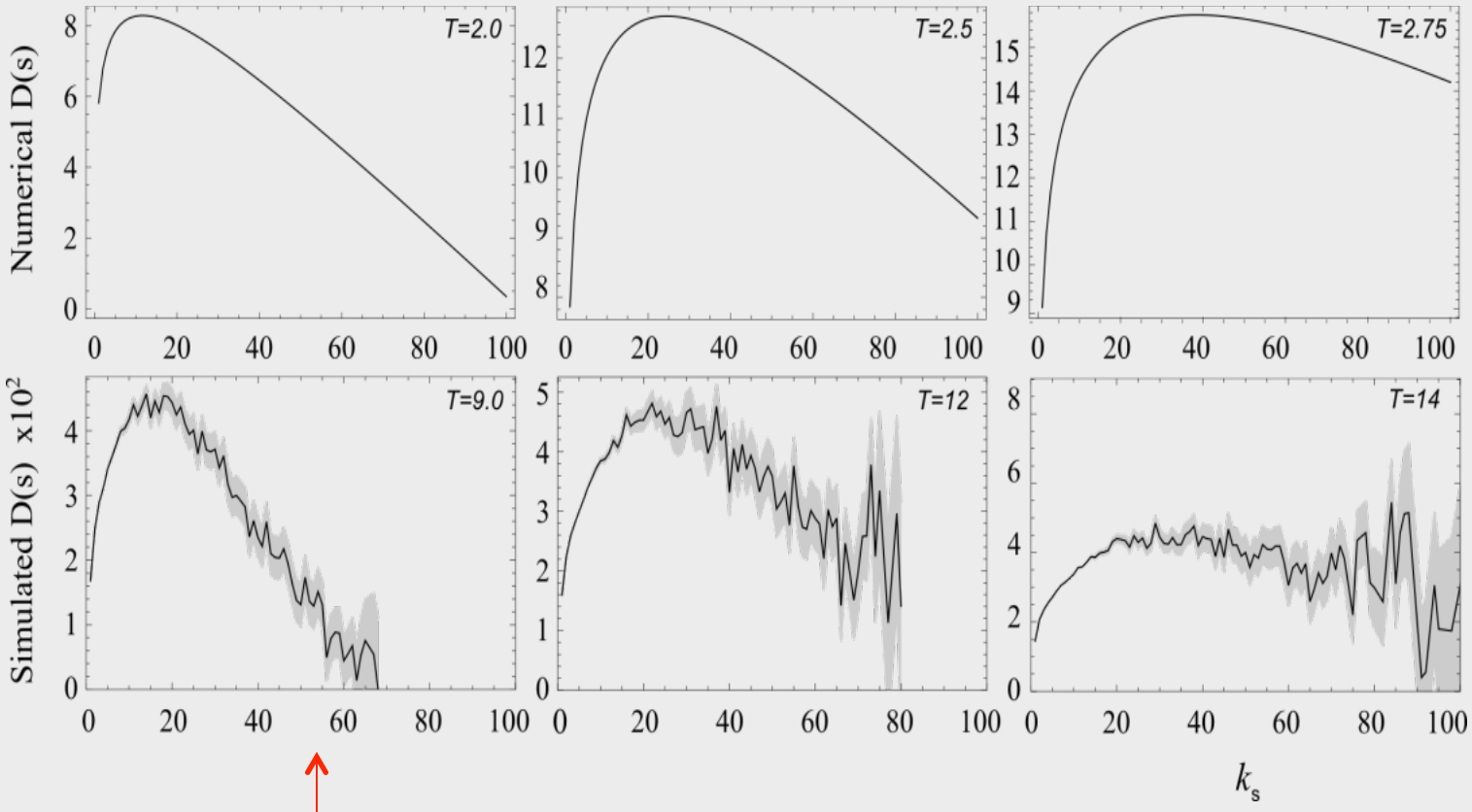
$$T(k) = \left\langle H(s_i^t) - H(s_i^t \mid s_j^{t+1}) \right\rangle_{k_j}.$$

$$\begin{aligned} H(s_i^t) &= -\sum_{q \in \Sigma} p(s_i^t = q) \log p(s_i^t = q) \\ &= -\sum_{q \in \Sigma^+} (1 - b_q^{-k}) \log(1 - b_q^{-k}) - \sum_{q \in \Sigma^-} b_q^{-k} \log b_q^{-k} \\ &= -\sum_{q \in \Sigma^+} (1 - b_q^{-k}) \log(1 - b_q^{-k}) + k \sum_{q \in \Sigma^-} b_q^{-k} \log b_q \\ &\approx k \sum_{q \in \Sigma^-} b_q^{-k} \log b_q \\ &= O(k \cdot x^{-k}). \end{aligned}$$

$$T(k+1) = \alpha \cdot T(k), \text{ where } \alpha \leq 1.$$

Results: analytical and numerical

Information dissipation time D(s) of a node s



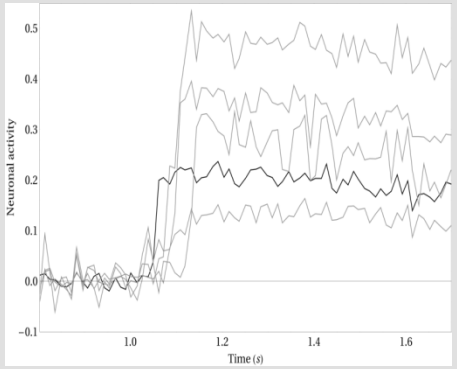
Number of interactions of a node

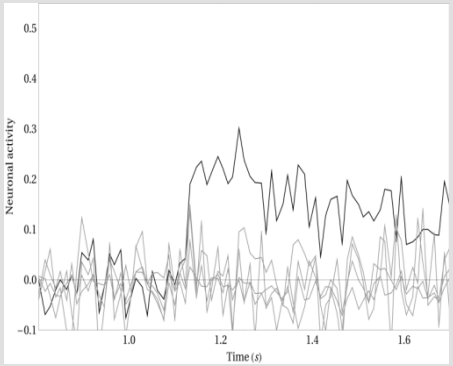
analytical proof: D(*s*) will eventually be a decreasing function of k_s

Qualitative evidence from experiments

Network of neurons cultured in a Petri dish

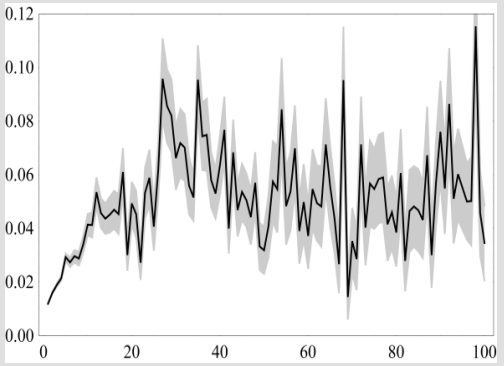
#1

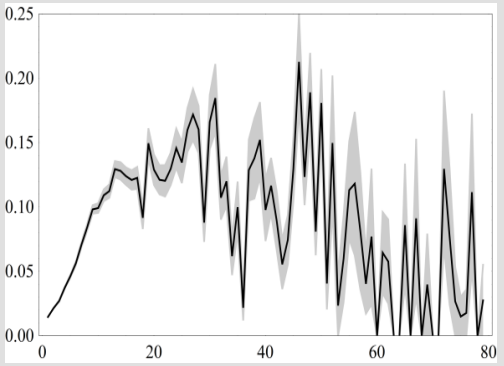




Social network of word-of-mouth marketing

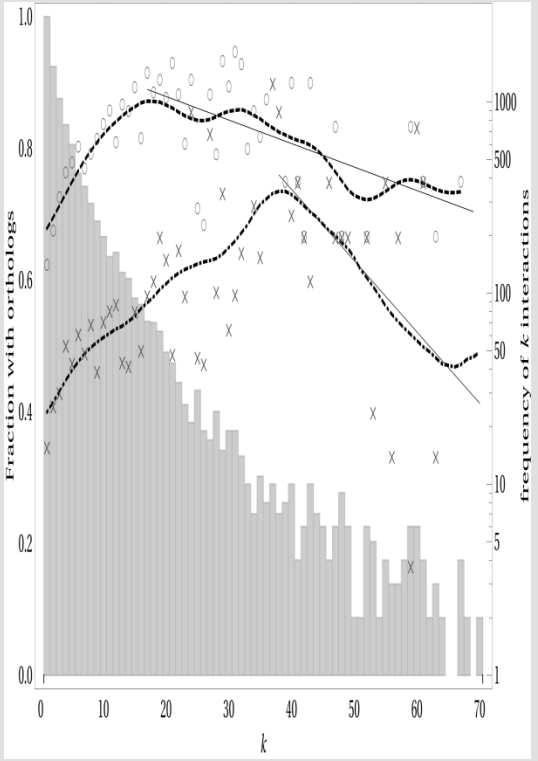
#2





Gene regulation network

#3



Applications of IDT and IDL

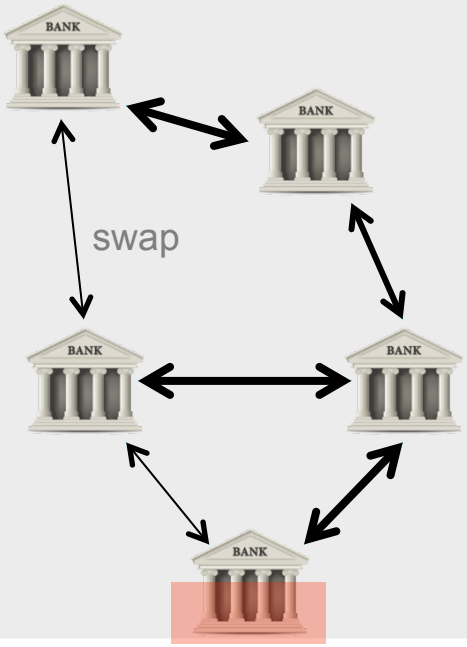
SCIENTIFIC REPORTS

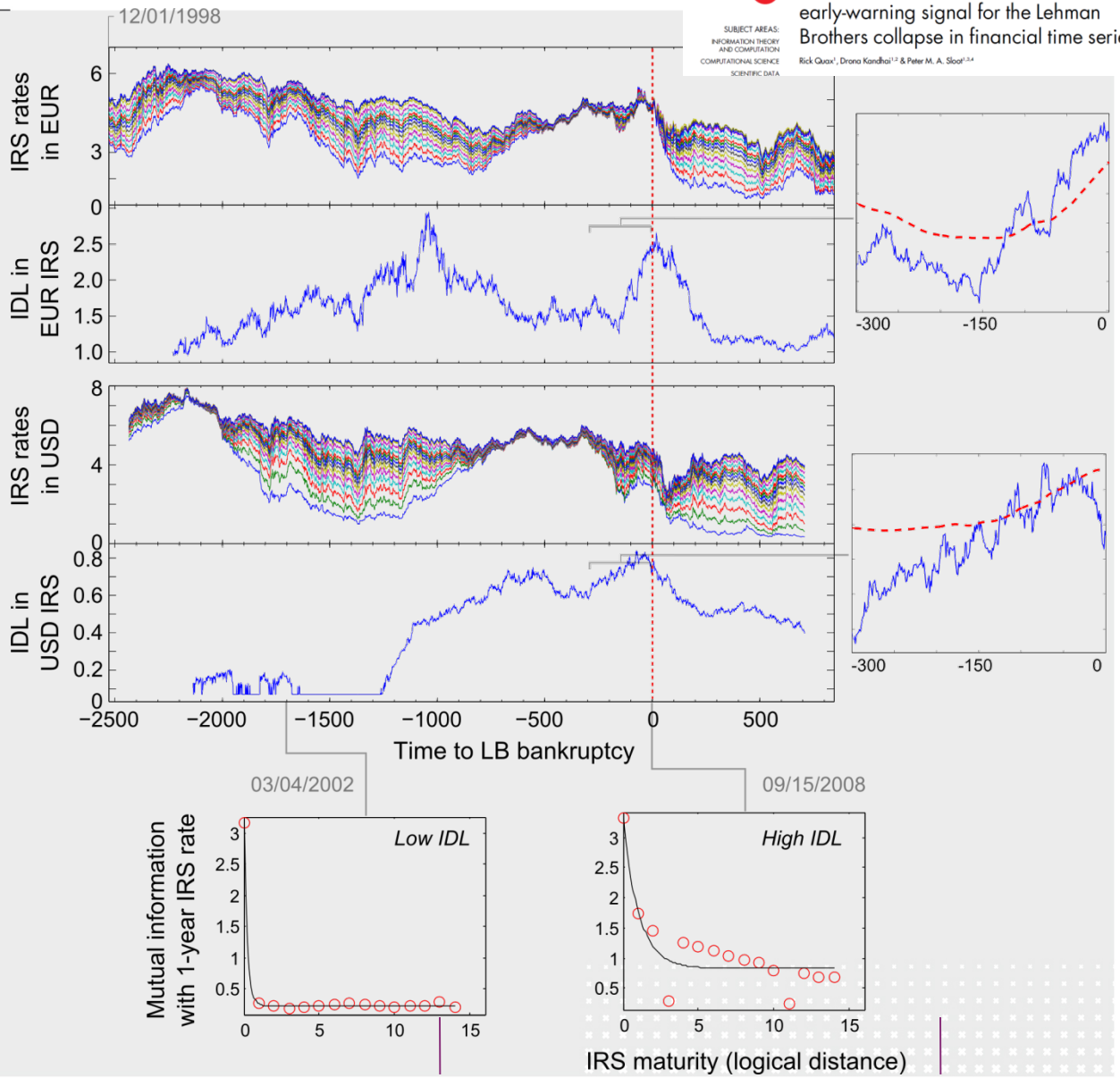
OPEN
SUBJECT AREAS: INFORMATION THEORY AND COMPUTATION; COMPUTATIONAL SCIENCE; SCIENTIFIC DATA

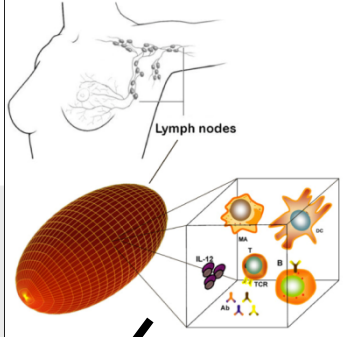
Information dissipation as an early-warning signal for the Lehman Brothers collapse in financial time series

Rick Quax[1], Drona Kandhai[1,2] & Peter M. A. Sloot[1,3,4]

Leading indicator in financial markets



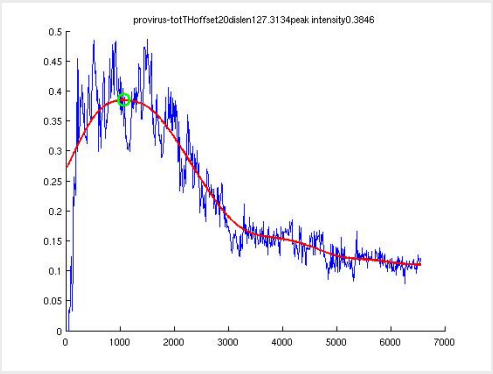




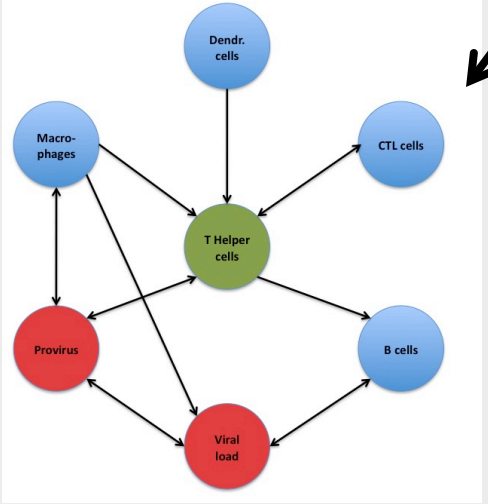
Cell types in immune response and their interactions

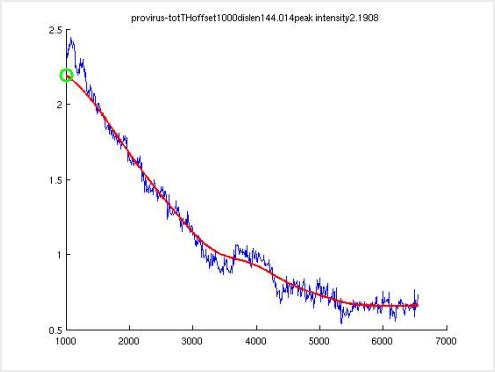
Susceptibility of HIV immune response to perturbation

$$I\left(\text{provirus}(t_0);\text{CD4}(t_0+t)\right)$$



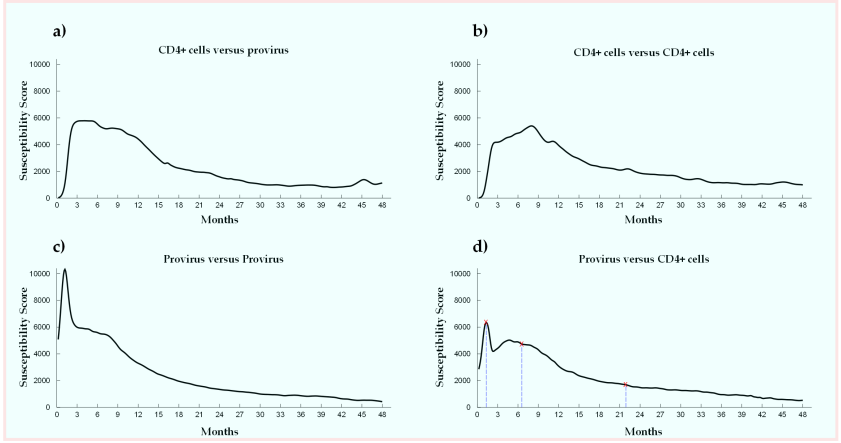
Agent-based simulations





IDT

Susceptibility of immune system



Acknowledgements

EU FP7 projects:

DynaNets

Sophocles

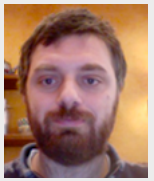
topdrim

Collaborators:

Peter Sloot

Emiliano Mancini

Andrea Apolloni



Drona Kandhai