Correlation between topological complexity and entropy in Idiotypic Network

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Abstract

In the present work we intend to study the correlation between the topology of the *idiotypic network* among antibodies and its Shannon entropy.

The idiotypic network hypothesis, formulated by Niels Jerne in 1974, postulates that the immune response may be regulated by responses to idiotypes, unique determinants originally described on B cell, and now also on T-cell receptors [3]. We are interesting in the study of the *idiotypic cascade*: The antibodies (called Ab_i) elicited directly by the antigen are a new protein for all practical purposes; they elicit the production of new antigens (Ab_2), which induce Ab_3, which induce Ab_4, which induce Ab_5, and so on $\cdots$. After a first transient in which the network responds to the antigens, it reaches a *metastable* condition in which only some antibodies alive. The metastable condition represents a crucial point for the immune system: the memory state. Actually, one of the most promising tool for the study of complex networks is *topological data analysis* (TDA, for short). TDA is a set of technique derived from *computational topology*. The main feature of computational topology is *persistent homology*: its basic aim is to measure the lifetime of certain topological properties of a simplicial complex when simplices are added to the complex or removed from. Persistent homology detects the topological features and provides both a global summarization using the Betti numbers, β_i, and a parametric representation of their birth and death with the *barcode*. *Clique Weight Rank Persistent Homology* (CWRPH) is a recent technique of TDA that can be used for the study of complex networks, briefly it does a filtration of the weighted cliques and then applies the persistent homology to the clique complexes. We used *jHoles* for the computation of CWRPH [5]. If TDA is a useful tool for the qualitative study of

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local an global connectivity of the network, entropy of immune cell distributions can provide a reliable tool for the quantitative study of the dynamics during the evolution of the idiotypic cascade: especially entropy can highlight the differences between the state in which system reacts to the external stimuli with its memory state.

In our work we investigate the correlation between topological complexity (1) with Shannon entropy (2):

$$C = \Sigma_{i=0}^{i=n} \beta_i \tag{1}$$

$$H(x) = -\Sigma_{i=1}^{i=m} p_i log_2 p_i \tag{2}$$

We simulated the idiotypic network using the agent-based model **C-IMMSIM** simulator [1]. The idiotypic network has been obtained starting from $10mL$ of antigens and $2^{12} = 4096$ possible antibodies with a period of 1095 time steps (1 year). For each tick:

- we computed the Shannon entropy. Where the probability associated to each antibody is a frequentist probability
- we built an *antibodies weighted network*, where the weight function is a coexistance formula based on the similarities among antibodies and their concentrations
- for each network we used jHoles to compute the topological complexity

Conclusions: During the reponse phase to the antigens the network has both the highest value of entropy and topological complexity. That means the network is completely unorganized, its structure has an high porosity (elevated number of β_1) then the system has not memory. In the metastable condition the entropy and the topological complexity have both an approximately constant value greather than zero but significantly less than their previous levels. The correlations between entropy and topological complexity is still under study but preliminary results indicate a logarithmic law. In the future we will investigate the relation with other entropy measurements [2].

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