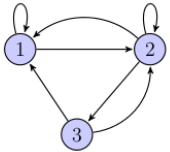
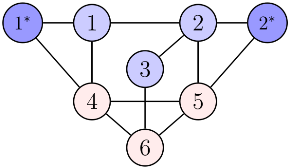
Persistent homology for digraphs

- Persistent homology offers a new approach to study complex networks.
- An undirected graph $G = (V, E)$ can be considered as a one dimensional simplicial complex where V is the set of the zero simplex and E the set of the one-simplex.





- A cycle is a sequence of nodes $i_1, i_2, \dots, i_N$ cyclically ordered such that
$$\{i_k, i_{k+1}\} \in E$$
for any $k = 1, 2, \dots, N$, with $i_{N+1} = i_1$.
- Taken a digraph G, we construct an undirected graph $\tilde{G}$ performing a duplication of any node in order to separate the source and the target of the original double edges and substituting any loop $e_{i,i}$ with a new node denoted (i^*).

Filtration

$\omega_{ij} > \epsilon$

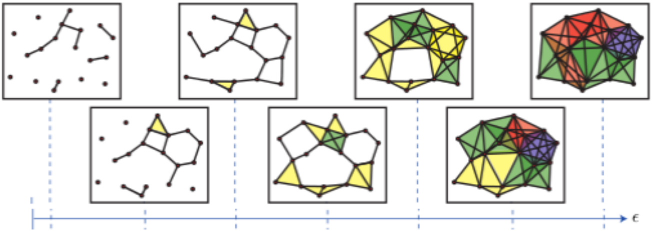


Figure from GHRIST (2008)

- Consider the thresholded graph $\tilde{G}_\epsilon$. This is the unweighted graph with links of weight larger than ϵ.
- For each graph $\tilde{G}_\epsilon$, apply Homology and calculate the cycles.
- For any cycle g, we have the birth index β_g corresponding to the threshold value at which a given generator is found and coinciding with the smallest weight of the network at the given threshold scale.

Conclusion

- We offer a method for analysing homology cycles and distinguish among their relevance.
- The construction developed allows the application of persistent homology to the study of a large variety of systems like the ones whose dynamics can be modelled by Markov chain models.
- Further, this analysis can be extended to higher dimensional homology groups in future.

Thanks for your attention!

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